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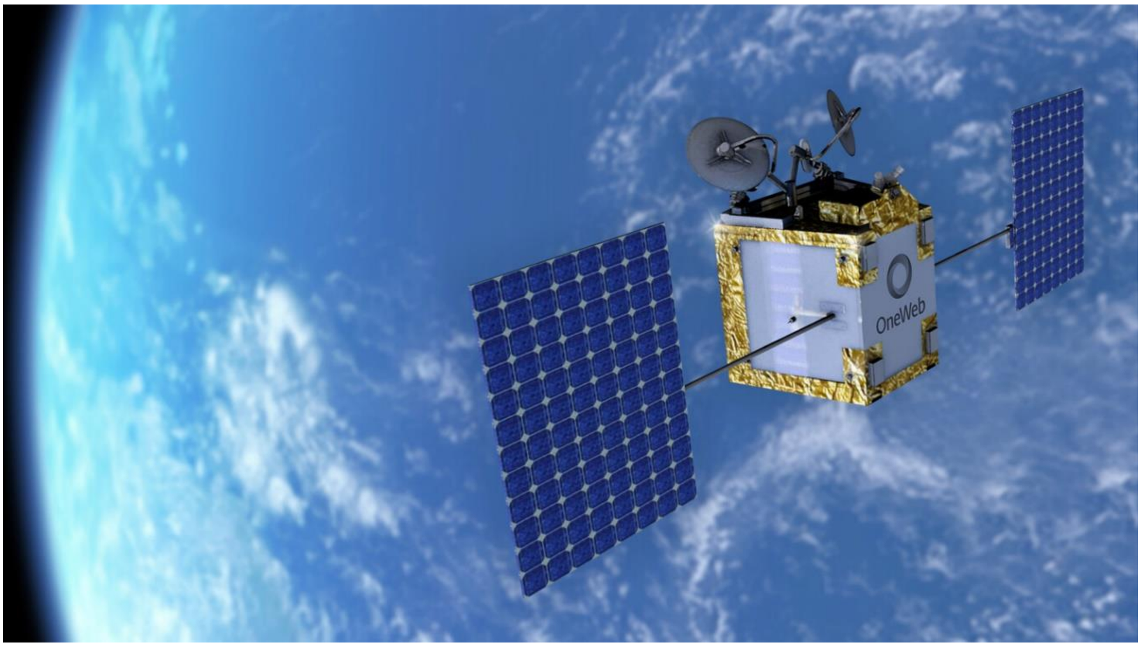

PRACTICE 1: Ground track of satellites

GROUP 16
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Introduction:

In this report we will plot the ground track from all the GPS constellation satellites as well as the ONEWEB-0410 satellite, in practice any satellite could be plotted knowing its Keplerian elements.

Some elements such as the `almanac.m` functions are already provided; therefore, they will be ignored during the explanation.

Data is obtained from http://celestrak.com/GPS/almanac/Yuma/2018/ as well as http://celestrak.com/NORAD/elements/supplemental/ for the TLE information.

Once all these information has been processed, we will be in position to plot it converting the ECEF to Lat Long coordinates.

The approach that has been taken is the following:

First of all, we focused on one GPS satellite plotting, then we scaled up to the entire constellation, and finally the function was copied and adapted to deal with TLE format.

GPS ephemeris:

An ephemeris is a table set of astronomical data which allows to compute the position of a spaceship or a celestial body having a time reference, in satellite position systems is used jointly with a time delta to compute the position of a user by trilateration. As stated before, this is done in the code calling the *almanac.m* function which returns a matrix `Eph` where every row is a satellite and, in every column, a different astronomical data is stored.

Now every fill will be discussed:

1. ID: PRN (Pseudo Random Noise) identification of the SVN (Space Vehicle NAVSTAR).

2. Health: Indicates the operational state of the SV (Space Vehicle). The value "000" means that the SV is usable.

3. Eccentricity: Shows the amount of the orbit deviation from a circular orbit. It is the distance between the foci divided by the length of the semi-major axis in GPS~0.

4. Time of Applicability (ToA): Time instant (seconds within the GPS week) for which the almanac has been computed (ephemeris epoch).

5. Orbital inclination: Angle of the SV orbit plane with respect to the equator in radians. The SV ground track will not rise above approximately 55º of latitude.

6. Rate of Right Ascension: Rate of change of the angle of right ascension in rad/s.

7. Square Root of Semi-Major Axis (SQRT(A)): Distance from the centre of the ellipse to the point of apogee for the point of perigee.

8. Right Ascension at GPS week epoch: Geographic longitude (in radians with respect to the Greenwich meridian) of the ascending node of the orbit at the GPS week epoch (Saturday to Sunday midnight).

9. Argument of Perigee at ToA: Angular measurement (in radians) along the orbital path measured from the ascending node to the point of perigee. It is measured in the direction of the SV's motion.

10. Mean Anomaly at ToA: Angle (radians) travelled past the perigee at ToA. When the SV has passed perigee and heading towards apogee, it is positive. After the point of apogee, it is negative to the point of perigee.

11. Af (0): SV clock bias in seconds.

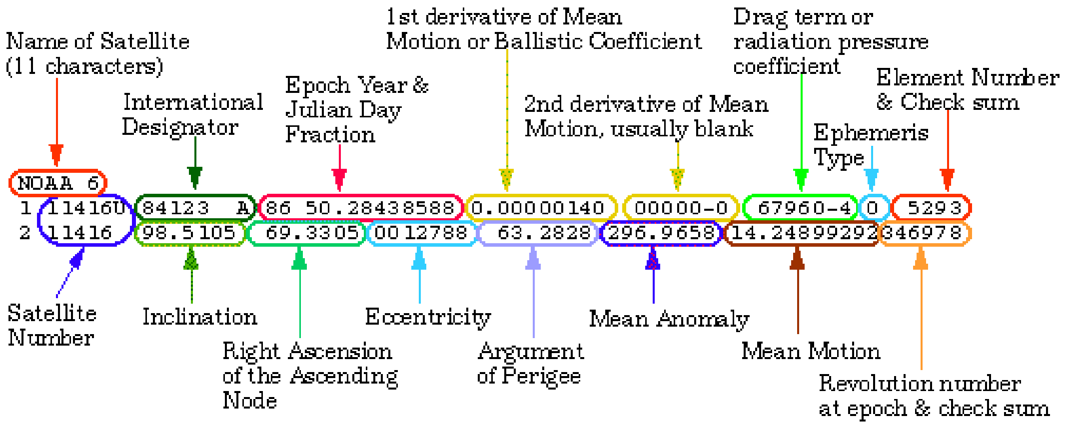
12. Af (1): SV clock Drift in seconds per seconds.

13. Week: GPS week number (0000-1023) limited to 10bits, once 1023 week has been reached, it is reset back to 0000.

TLE datagram:

TLE stands for Two Line Element set. It is a different way of providing the astronomical data bellow an example from the ISS and a diagram can be found:

```
ISS (ZARYA)
 1 25544U 98067A 14273.50403866 .00012237 00000-0 21631-3 0 1790
 2 25544 51.6467 297.5710 0002045 126.1182 27.2142 15.50748592907666
```



All radial units are given in degrees and the eccentricity is given in relation to ten to the power of seven.

ECEF coordinates

Earth Centered, Earth Fixed (ECEF) Cartesian coordinates where all the centre is taken as the Earth's centre, the system rotates solitary with the Earth z points the North pole, x points to the Equator and crosses the Greenwich meridian.

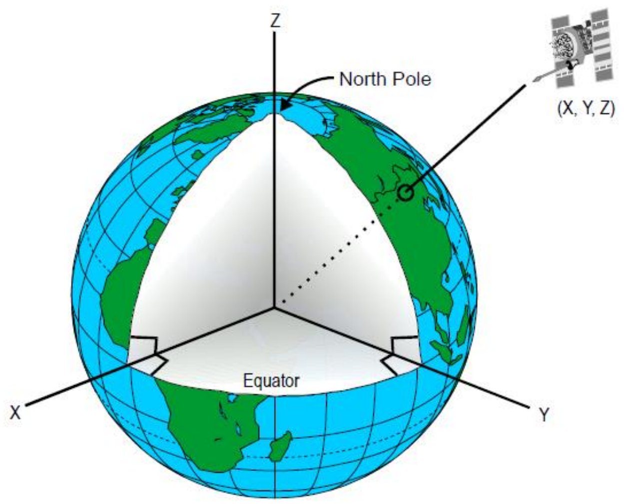


Fig. 1 ECEF system

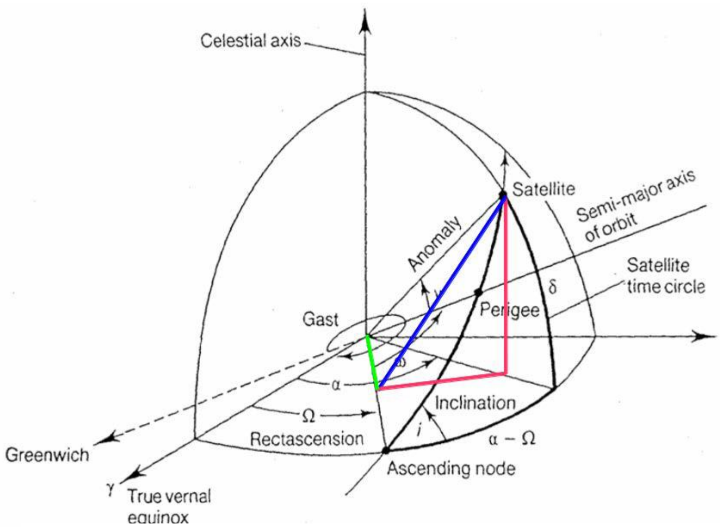


Fig. 2Kepplerian elements

To convert the Keplerian data to Cartesian, we must simply follow the algorithm provided in the SoW.

The variables used will be the following ones:

a	Orbit major semi axis [m]
i_0	Orbit inclination [rad]
e	Orbit eccentricity
Ω_0	Longitude of the ascending node (AN) at the ToA [rad]
$\dot{\Omega}_0$	Rate of change of the right ascension of the AN [rad/s]
ω	Argument of the perigee at ToA [rad]
M_0	Mean anomaly at ToA [rad]
n	Satellite mean motion [rad/s]
dt	Elapsed time from the ToA [s] (negative if the ToA is in the future)

Now we will comment each of the step of the algorithm:

1. **Time** between ToA and the current time:

$$t_k = t - t_o$$

2. **Semi-major axis**:

$$a = \left(\sqrt{a}\right)^2$$

3. **Mean motion:**

$$n = \sqrt{\frac{G \cdot M}{a^3}}$$

4. **longitude of the ascending node at the ToA:**

$$\Omega_0 = \Omega'_0 - \dot{\Omega}_e \cdot t_0$$

5. **Mean anomaly:**

$$M_k = M_o + n \cdot t_k$$

6. **Eccentric anomaly:**

$$M_k = E_k - e \cdot sin\,(E_k)$$

$$E_k = M_k + e \cdot sin\,(E_k)$$

This step is an iterative one where the values are recalculated until ensuring $|E_k(n) - E_k(n-1)| < 10^{-8}$

7. **True anomaly:**

$$sin(v_k) = \frac{\sqrt{1-e^2} \cdot sin(E_k)}{1 - e \cdot cos(E_k)} \qquad cos(v_k) = \frac{cos(E_k) - e}{1 - e \cdot cos(E_k)}$$

8. **Latitude argument:**

$$u_k = v_k + \omega$$

9. **Orbital radius:**

$$r_k = a \cdot (1 - e \cdot cos(E_k))$$

10. **Longitude of the ascending node:**

$$\Omega_k = \Omega_0 + \dot{\Omega}_0 \cdot dt - \dot{\Omega}_e \cdot dt$$

11. **x coordinate:**

$$x_p = r_k \cdot cos(u_k)$$

12. **y coordinate:**

$$y_p = r_k \cdot sin(u_k)$$

Now we are in position to translate these coordinates to ECEF:

13. **ECEF x-coordinate:**

$$x = x_p \cdot cos(\Omega_k) - y_p \cdot cos(i_0) \cdot sin(\Omega_k)$$

14. **ECEF y-coordinate:**

$$y = x_p \cdot sin(\Omega_k) + y_p \cdot cos(i_0) \cdot cos(\Omega_k)$$

15. **ECEF z-coordinate:**

$$z = y_p\, sin(i_0)$$

TLE system:

For TLE, the procedure is very similar, we only must translate the given data to fit the algorithm, this is done by using `time2toa.m` function which returns the time like "esec" and the GAST which is used in step 4, also is important to bear in mind that the rate of change of the right ascension in TLE is zero.

Plotting the results:

To plot the results, we must change the ECEF system to Lat/Long coordinates, this is done by following an algorithm:

1. $r = \sqrt{x^2 + y^2}$

2. $E^2 = a^2 - b^2$

3. $F = 54b^2z^2$

4. $G = r^2 + (1 - e^2)z^2 - e^2E^2$

5. $c = \frac{e^4Fr^2}{G^3}$

6. $s = \sqrt[3]{1 + c + \sqrt{c^2 + 2c}}$

7. $P = \frac{F}{3\left(s+\frac{1}{s}+1\right)^2G^2}$

8. $Q = \sqrt{1 + 2e^4P}$

9. $r_o = -\frac{Pe^2r}{1+Q} + \sqrt{\frac{1}{2}a^2\left(1 + \frac{1}{Q}\right) - \frac{P(1-e^2)z^2}{Q(1+Q)} - \frac{1}{2}Pr^2}$

10. $U = \sqrt{z^2 + (r - r_oe^2)^2}$

11. $V = \sqrt{z^2(1 - e^2) + (r - r_oe^2)^2}$

12. $z_o = \frac{b^2z}{aV}$

13. $h = U\left(1 - \frac{b^2}{aV}\right)$

14. $\phi = arctg\left(\frac{z+e'^2z_o}{r}\right)$

15. $\lambda = arctg\left(\frac{y}{x}\right)$

In order to use c and b, we are provided with the following identities:

$$a^2 = b^2 + c^2$$

Datum	a	e^2
WGS-84	6378137 m	0.00669437999014

Results:

GPS constellation:

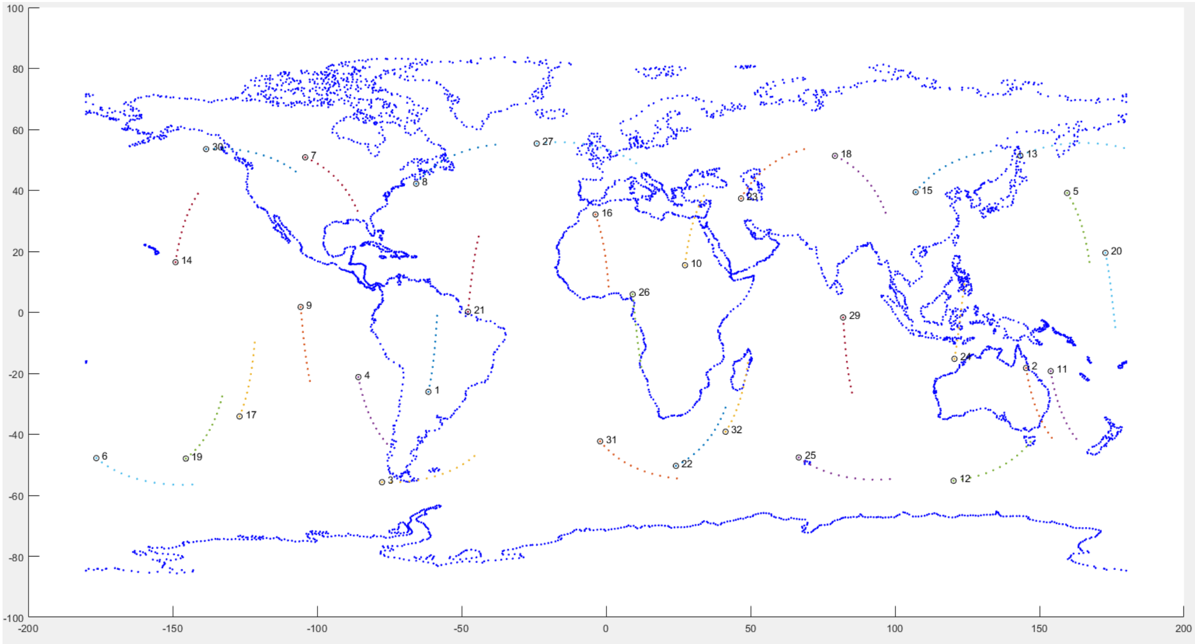


Fig. 3GPS track computation

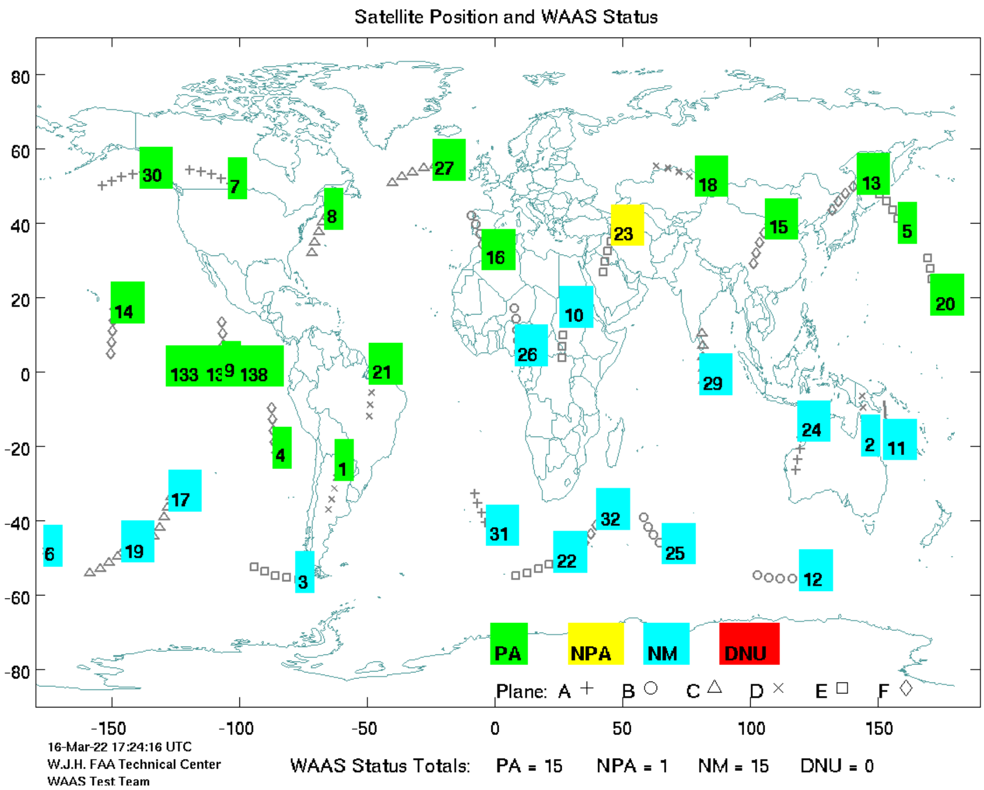


Fig. 4http://www.nstb.tc.faa.gov/rt_waassatellitestatus.htm GPS tracker

Single satellite 24h:

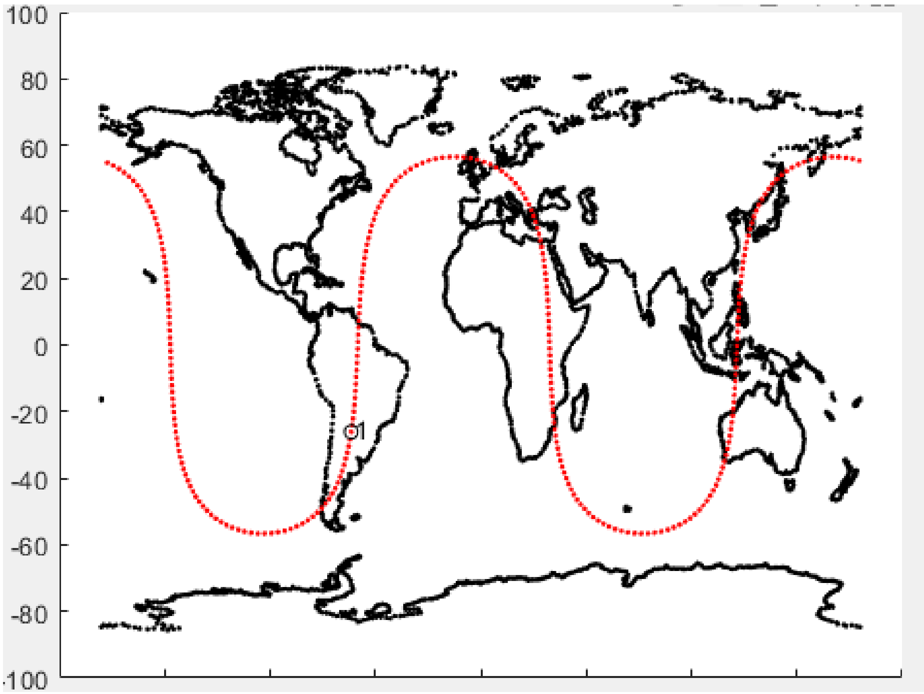


Fig. 6 GPS satellite 1 one day track

ONEWEB

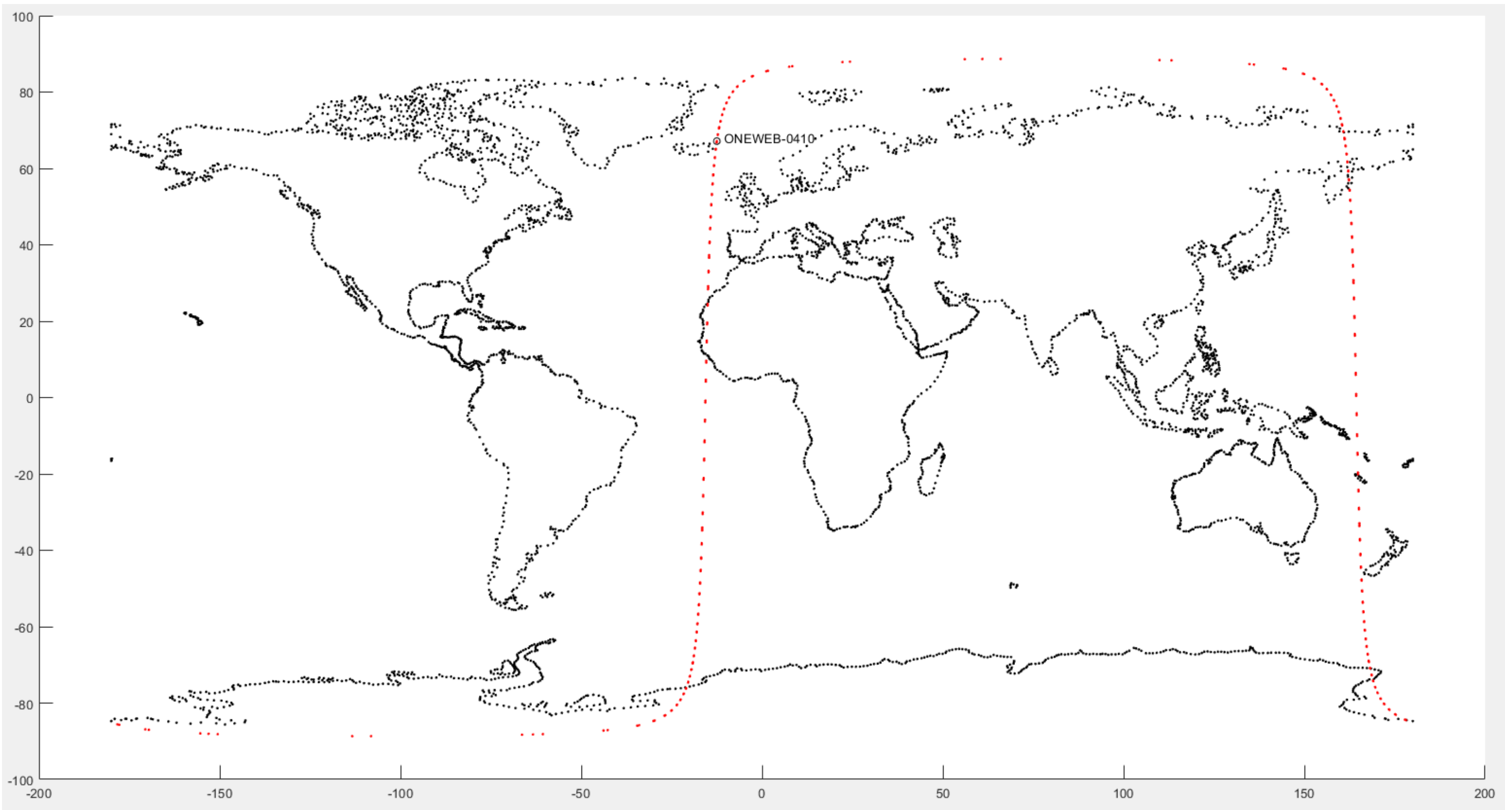


Fig. 5 ONEWEB-0410 track

Conclusions:

We have successfully archived our main goals, the most difficult part was when plotting the first satellite, from that point we only had to modify the code in order to answer the other requirements from the statement of work.

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